Appendix 2

Necessary Ordered Logit Formulae

The following discussion has been adapted from Greene's (1993, Chapter 21.7.2) discussion of ordered data.

Calculation of Probabilities

Using the β's and μ estimated from the ordered logit regression for a given autonomy index, the probability of being in the low, medium or high category of the index can be evaluated such that

$$Prob(y = low) = \Lambda\,(-\beta' X) \tag{1}$$

$$Prob(y = medium) = \Lambda\,(\mu - \beta' X) - \Lambda\,(-\beta' X\,) \tag{2}$$

$$Prob(y = high) = 1 - \Lambda\,(\mu - \beta' X) \tag{3}$$

where Λ is the cumulative density function of the logistic distribution given by:

$$\Lambda\,(\beta' X) = \frac{e^{\beta' X}}{1 + e^{\beta' X}} \quad . \tag{4}$$

These probabilities will necessarily add to 1.

Conversion of Coefficients into Probability Changes

A. For continuous variables

The effect of a marginal change in a continuous regressor *i* on the probability of being in each of the three categories of the dependent variable is given by:

$$\frac{\delta\ Prob(y = 0)}{\delta X} = -\kappa(\beta' X)\beta_i, \tag{5}$$

$$\frac{\delta\ Prob(y = 1)}{\delta X} = (\kappa(-\beta' X) - \kappa(\mu - \beta' X))\beta_i, \tag{6}$$

$$\frac{\delta\ Prob(y = 2)}{\delta X} = \kappa(\mu - \beta'X)\beta_i, \tag{7}$$

where

$$\kappa = \frac{d\Lambda[\beta'X]}{d(\beta'X)} = \frac{e^{\beta'X}}{(1 + e^{\beta'X})^2} \quad . \tag{8}$$

Note: The sum of the marginal effects of a given regressor on the probabilities of being in the three categories must add to 0. For each calculation the values of the other regressors are held at their means.

B. For dummy variables

The effect of a dummy variable is calculated by estimating the probability of being in each category of the dependent variable using equations 1-3, at the two values of the dummy variable (0 and 1). The absolute difference in probability for each category is due to the dummy taking the value 1 instead of 0. These differences must add to 0. The percent difference in probability is then calculated as the change in probability divided by the probability when the dummy value is 0. The percent difference need not add to 0.